CBCS SCHEME

USN | | | | | | | | | | |

18CS36

Third Semester B.E. Degree Examination, July/August 2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions.***

1 a. Define the following with an example for each i) Proposition ii) Tautology iii) Contradiction. **(06 Marks)**

b. Establish the validity of the argument:

$$\begin{array}{l} p \rightarrow q \\ q \rightarrow (r \wedge s) \\ \neg r \vee (\neg t \vee u) \\ \neg r \vee (\neg t \vee u) \\ \underline{p \wedge t} \\ \therefore U \end{array}$$

(09 Marks)

c. Determine the truth value of the following statements if the universe comprises of all non zero integers:

i) $\exists_x \exists_y [xy = 2]$

ii) $\exists_x \forall_y [xy = 2]$

iii) $\forall_x \exists_y [xy = 2]$

iv) $\exists_x \exists_y [(3x + y = 8) \wedge (2x - y) = 7]$

v) $\exists_x \exists_y [(4x + 2y = 3) \wedge (x - y = 1)]$ **(05 Marks)**

2 a. Using truth table, prove that for any three propositions p, q, r $[p \rightarrow (q \wedge r)] \Leftrightarrow [(p \rightarrow q) \wedge (p \rightarrow r)]$. **(08 Marks)**

b. Prove that for all integers 'k' and 'l', if k and l both odd, then $k + l$ is even and kl is odd by direct proof. **(06 Marks)**

c. If a proposition has truth value 1, determine all truth values arguments for the primitive propositions p, r, s for which the truth value of the following compound proposition is 1.
$[q \rightarrow \{ (\neg p \vee r) \wedge \neg s\}] \wedge \{\neg s \rightarrow (\neg r \wedge q)\}$ **(06 Marks)**

3 a. Prove by mathematical induction for every positive integer 8 divides $5^n + 2 \cdot 3^{n-1} + 1$. **(06 Marks)**

b. For the Fibonacci sequences $F_0, F_1, F_2 \ldots$. Prove that $F_n = \frac{1}{\sqrt{5}}\left[\left(\frac{1+\sqrt{5}}{2}\right) - \left(\frac{1-\sqrt{5}}{2}\right)^n\right]$. **(06 Marks)**

c. Find the coefficient of :

i) $x^9 y^3$ in the expansion of $(2x - 3y)^{12}$

ii) x^{12} in the expansion $x^3(1 - 2x)^{10}$ **(08 Marks)**

4 a. Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. **(06 Marks)**

b. How many positive integers 'n' can we form using the digits 3, 4, 4, 5, 5, 6, 7 if we want 'n' to exceed 5,000,000. **(08 Marks)**

c. Find the number of distinct terms in the expansion of $(w + x + y + z)^{12}$. **(06 Marks)**

5 a. i) Let $f: R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5, & \text{for } x > 0 \\ -3x + 1, & \text{for } x \leq 0 \end{cases}$$

Determine: $f\left(\frac{5}{3}\right)$, $f^{-1}(3)$, $f^{-1}([-5, 5])$ **(04 Marks)**

ii) Prove that if 30 dictionaries contain a total of 61,327 pages, then at least one of the dictionary must have at least 2045 pages. **(02 Marks)**

b. Prove that if $f : A \to B$ and $g : B \to C$ are invertible functions then $gof : A \to C$ is an invertible function and $(gof)^{-1} = f^{-1} \circ g^{-1}$. **(06 Marks)**

c. Let A = {1, 2, 3, 4, 5}. Define a relation R on $A \times A$ by $(x_1, y_1)\, R(x_2, y_2)$ if and only if $x_1 + y_1 = x_2 + y_2$.

i) Determine whether R is in equivalence relation on $A \times A$.

ii) Determine equivalence classes [(1, 3)], [(2, 4)]. **(08 Marks)**

6 a. Let A = {a, b, c, d} and B = {1, 2, 3, 4, 5, 6}

i) How many functions are there from A to B? How many of these are one-to-one? How many are onto?

ii) How many functions are there from B to A? How many of these are one-to-one? How many are onto? **(06 Marks)**

b. Let A = {1, 2, 3, 4, 6, 12}. On A define the relation R by aRb if and only if "a divides b".

i) Prove that R is a partial order on A

ii) Draw the Hasse diagram

iii) Write down the matrix of relation. **(08 Marks)**

c. Define partition of a set. Give one example Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}{e}} of A. Find the equivalent relation inducing this partition. **(06 Marks)**

7 a. Out of 30 students in a hostel; 15 study History, 8 study Economics and 6 study Geography. It is known that 3 students study all these subjects. Show that 7 or more students study none of these subjects. **(06 Marks)**

b. Five teachers T_1, T_2, T_3, T_4, T_5 are to made class teachers for five classes C_1, C_2, C_3, C_4, C_5 one teacher for each class. T_1 and T_2 do not wish to become the class teachers for C_1 or C_2, T_3 and T_4 for C_4 or C_5 and T_5 for C_3 or C_4 or C_5. In how many ways can the teachers be assigned work without displeasing any teacher? **(08 Marks)**

c. Solve the recurrence relation $a_n - 6a_{n-1} + 9a_{n-2} = 0$ for $n \geq 2$. **(06 Marks)**

8 a. Solve the recurrence relation $a_0 - 3a_{n-1} = 5 \times 3^n$ for $n \geq 1$ given that $a_0 = 2$. **(06 Marks)**

b. Let a_n denote the number of n-letter sequences that can be formed using letters A, B and C, such that non terminal A has to be immediately followed by B. Find the recurrence relation for a_n and solve it. **(06 Marks)**

c. Find the number of permutations of English letters which contain exactly two of the pattern car, dog, pun, byte. **(08 Marks)**

9 a. Define a complement of a simple graph. Let G be a simple graph of order n. If the size of G is 56 and size of $\overline{G}$ is 80, what is n? **(06 Marks)**

b. Prove that is every graph, the number of vertices of odd degree is even. **(08 Marks)**

c. Prove that a connected graph G remains connected after removing an edge e from G if and only if e is a part of some cycle in G. **(06 Marks)**

10 a. Define graph isomorphism and isomorphic graphs. Determine whether the following graphs are isomorphic or not. **(06 Marks)**

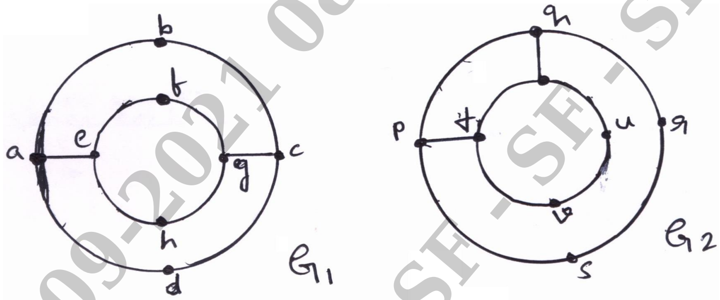

Fig.Q.10(a)

b. Prove that a tree with 'n' vertices has n – 1 edges. **(06 Marks)**

c. Define optimal prefix code. Obtain the optimal prefix code for the string ROAD is GOOD. Indicate the code. **(08 Marks)**

* * * * *